Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name : Topology-I

Subject Code : 5SC01MTC4 **Branch : M. Sc. (Mathematics)**

Semester : 1 **Date : 7 / 12 / 2015** **Time : 10:30 To 1:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Define: Discrete topology. **[2]**

b. Let $X = \{a, b, c, d\}$ and $\tau = \{X, \phi, \{a\}, \{a, b\}, \{a, b, c\}\}$ be a topology on X. Find open sets and closed set of τ on X. **[2]**

c. If $A = [0] \cup (1, 2)$ then find $\bar{A}$. **[1]**

d. Define: Hausdorff space. **[2]**

Q-2 **Attempt all questions** **(14)**

a) Show that a subspace of a regular space is regular. **[6]**

b) Define: Countable complement topology. Hence show that countable complement topology (τ_c) is a topology on X. **[6]**

c) If τ is a topology generated by the basis $\mathcal{B}$ then show that $\mathcal{B} \subset \tau$. **[2]**

OR

Q-2 **Attempt all questions** **(14)**

a) Let A, B and A_α denote subsets of a topological space X. Prove that $\overline{A \cup B} = \bar{A} \cup \bar{B}$. **[6]**

b) Show that the lower limit topology τ_l on R is strictly finer than the standard topology τ. **[6]**

c) Show that $[a, b]$ is closed in R. **[2]**

Q-3 **Attempt all questions** **(14)**

a) Let $X = R$ and $\mathcal{B} = \{(a, b)/a, b \in R\}$. Then show that $\mathcal{B}$ is a basis for a topology on R. **[5]**

b) Show that if Y is a subspace of X and $A \subset Y$, then the subspace topology on A as a subspace of Y is the same as the subspace topology on A as a subspace of X. **[5]**

	c)	If y is a point of the basis element $B(x,\epsilon)$ then show that there is a basis element $B(y,\delta)$ such that $B(y,\delta) \subset B(x,\epsilon)$.	[4]

OR

Q-3	a)	Show that if a space satisfies the second axiom of countability then it satisfies the first axiom of countability.	[5]
	b)	Show that T_3 −space is a T_2 −space.	[5]
	c)	Show that intersection of two topologies is also topology on X. What can you say about union of two topologies? Justify your answer.	[4]

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Define: Normal space.	[2]
	b.	State Urysohn lemma.	[2]
	c.	Define: Metric topology.	[2]
	d.	Define: Separable space.	[1]

Q-5		**Attempt all questions**	**(14)**
	a)	Let X be a metrizable space then show that if X is separable then X is second countable.	[5]
	b)	Every normal space is a regular space.	[5]
	c)	Every compact subset of a Hausdorff space is closed.	[4]

OR

Q-5	a)	If Y is a compact subset of a Hausdorff space X and $x_0 \notin Y$ then show that there exists open set U and V such that $x_0 \in U, Y \subset V$ and $U \cap V = \emptyset$.	[5]
	b)	Every closed subset of a compact space is compact.	[5]
	c)	Is the space R_l connected? Justify your answer.	[4]

Q-6		**Attempt all questions**	**(14)**
	a)	Define: Topology. Hence find all the topologies of the set $X = \{a, b, c\}$.	[5]
	b)	Show that $\tau_l = \{A \subset R / for\ each\ x \in A, \exists\ \epsilon > 0\ \ni [x, x+\epsilon) \subset A\} \cup \{\phi\}$ is a topology on $X = R$.	[5]
	c)	Let X be a set. Show that the collection $\tau_\infty = \{U / X - U\ \ is\ infinite\ or\ X - U = X\}$ is not a topology on X.	[4]

OR

Q-6	a)	Define: Subbasis. Let $X = R \cup \{-\infty\}, S = \{[-\infty, a)/a \in R\} \cup \{[b, \infty)/b \in R\}$ is a subbasis for a topology on X.	[5]
	b)	Show that $\mathcal{B} = \{(a \times b, a \times d)/b < d\}$ is a basis for the order topology on $R \times R$.	[5]
	c)	Show that a product of two Hausdorff spaces is Hausdorff.	[4]